

Measurement

Area of a sector

$$A = \frac{1}{2}r^2\theta$$

Arc length

$$l = r\theta$$

Surface area of a cone (lateral)

$$S = \pi rl$$

Surface area of a sphere

$$S = 4\pi r^2$$

Volume of a cone

$$V = \frac{1}{3}\pi r^2 h$$

Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

Volume of a pyramid

$$V = \frac{1}{3}Ah$$

Financial Mathematics

Future value of a compound interest investment

$$FV = PV(1 + r)^n$$

Present value of a compound interest investment

$$PV = FV(1 + r)^{-n}$$

Future value of an annuity (regular contributions)

$$FV = C \frac{(1 + r)^n - 1}{r}$$

Present value of an annuity

$$PV = C \frac{1 - (1 + r)^{-n}}{r}$$

Sequences and Series

Arithmetic sequence — nth term

$$T_n = a + (n - 1)d$$

Arithmetic series — sum of n terms

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}[2a + (n - 1)d]$$

Geometric sequence — nth term

$$T_n = ar^{n-1}$$

Geometric series — sum of n terms

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}$$

Infinite geometric series — limiting sum

$$S = \frac{a}{1 - r}, \quad |r| < 1$$

Logarithmic, Exponential and Indices

Change of base formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Logarithm product law

$$\log_a(xy) = \log_a x + \log_a y$$

Logarithm quotient law

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Logarithm power law

$$\log_a(x^n) = n \log_a x$$

Logarithm definition / inverse relationship

$$\log_a a^x = x \quad \text{and} \quad a^{\log_a x} = x$$

Index law — product

$$a^m \cdot a^n = a^{m+n}$$

Index law — quotient

$$\frac{a^m}{a^n} = a^{m-n}$$

Index law — power of a power

$$(a^m)^n = a^{mn}$$

Derivative of a^x (general exponential)

$$\frac{d}{dx}(a^x) = a^x \ln a$$

Trigonometry

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Area of a triangle

$$A = \frac{1}{2}ab \sin C$$

Exact trigonometric ratios — 30°, 45°, 60°

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Pythagorean identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

Compound angle — sine

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

Compound angle — cosine

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Compound angle — tangent

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double angle — sine

$$\sin 2A = 2 \sin A \cos A$$

Double angle — cosine (three forms)

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

Double angle — tangent

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Products to sums (product formulae)

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

t-formulae (half-angle substitution $t = \tan(\theta/2)$)

$$\sin \theta = \frac{2t}{1+t^2}, \quad \cos \theta = \frac{1-t^2}{1+t^2}, \quad \tan \theta = \frac{2t}{1-t^2}$$

Calculus — Differentiation

Power rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Product rule

$$\frac{d}{dx}(uv) = u'v + uv'$$

Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Derivative of $\sin x$

$$\frac{d}{dx}(\sin x) = \cos x$$

Derivative of $\cos x$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Derivative of $\tan x$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Derivative of e^x

$$\frac{d}{dx}(e^x) = e^x$$

Derivative of $\ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$$

Calculus — Standard Integrals

Power rule for integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Integral of $1/x$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Integral of e^{ax}

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

Integral of $\cos ax$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C$$

Integral of $\sin ax$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C$$

Integral of $\sec^2 ax$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + C$$

Integral giving arcsin (inverse sine)

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad |x| < a$$

Integral giving arctan (inverse tangent)

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

Integral of a^x (general exponential)

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Log-integral (reverse chain rule for $\ln$)

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

Probability and Statistics

Complement rule

$$P(\bar{A}) = 1 - P(A)$$

Addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Mean (expected value) of a discrete random variable

$$\mu = E(X) = \sum x_i P(X = x_i)$$

Variance and standard deviation of a discrete random variable

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

Binomial distribution

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Normal distribution and z-score

$$z = \frac{x - \mu}{\sigma}$$

Combinatorics (Ext 1)

Permutations (ordered selections)

$${}_n P_r = \frac{n!}{(n-r)!}$$

Combinations (unordered selections)

$$\binom{n}{r} = {}_n C_r = \frac{n!}{r!(n-r)!}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

Ext 1 — Further Calculus and Functions (Ext 1)

Derivative of $\sin^{-1} x$

$$\frac{d}{dx} \left(\sin^{-1} \frac{x}{a} \right) = \frac{1}{\sqrt{a^2 - x^2}}$$

Derivative of $\cos^{-1} x$

$$\frac{d}{dx} \left(\cos^{-1} \frac{x}{a} \right) = -\frac{1}{\sqrt{a^2 - x^2}}$$

Derivative of $\tan^{-1} x$

$$\frac{d}{dx} \left(\tan^{-1} \frac{x}{a} \right) = \frac{a}{a^2 + x^2}$$

Integration by substitution

$$\int f(g(x)) g'(x) dx = F(g(x)) + C$$

Ext 1 — Vectors (2D) (Ext 1)

Magnitude of a vector

$$|\mathbf{u}| = \sqrt{x^2 + y^2}$$

Dot product (scalar product)

$$\mathbf{u} \cdot \mathbf{v} = x_1x_2 + y_1y_2 = |\mathbf{u}||\mathbf{v}| \cos \theta$$

Scalar projection of $\mathbf{u}$ onto $\mathbf{v}$

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}$$

Ext 1 — Projectile Motion (Ext 1)

Horizontal displacement

$$x = V \cos \alpha \cdot t$$

Vertical displacement

$$y = V \sin \alpha \cdot t - \frac{1}{2}gt^2$$

Range on a horizontal plane

$$R = \frac{V^2 \sin 2\alpha}{g}$$

Ext 2 — Complex Numbers (Ext 2)

Modulus-argument (polar) form

$$z = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

Multiplication of complex numbers in polar form

$$r_1 \operatorname{cis} \theta_1 \times r_2 \operatorname{cis} \theta_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

De Moivre's theorem

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Ext 2 — Further Integration (Ext 2)

Integration by parts

$$\int u \, dv = uv - \int v \, du$$

Partial fractions

$$\frac{P(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

t-substitution for integrals ($t = \tan(x/2)$)

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2}{1+t^2} dt$$

Ext 2 — Vectors in 3D, Mechanics and SHM (Ext 2)

3D vector magnitude

$$|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$$

3D dot product and angle

$$\mathbf{u} \cdot \mathbf{v} = x_1x_2 + y_1y_2 + z_1z_2 = |\mathbf{u}||\mathbf{v}| \cos \theta$$

Simple harmonic motion — equation

$$\ddot{x} = -n^2x$$

SHM — displacement, velocity, acceleration solutions

$$x = a \cos(nt + \alpha) \quad \text{or} \quad x = a \sin(nt + \alpha)$$

SHM — velocity in terms of displacement

$$v^2 = n^2(a^2 - x^2)$$

Resisted motion — Newton's second law form

$$m\ddot{x} = F_{\text{net}}$$

