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HSC 2026 · PREDICTED PRACTICE PAPER

2026 HSC Mathematics Extension 2

Answers & marking notes

Paper total: 100 marks

Sit the paper first. These are worked answers with the marking notes behind them — what earns the mark, and the traps the marking centre has flagged in past years. Where a note names a band, it describes what a top-band response does, not a guaranteed mark.

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Answers & marking notes

Section I — correct responses

Q	1	2	3	4	5	6	7	8	9	10
Answer	B	C	D	C	B	A	A	D	B	C

Brief reasons:

1. Contrapositive negates **and swaps**: "not multiple of 3 $\Rightarrow$ not multiple of 6". A is the converse, C the inverse, D a (false) conjunction. **B**
2. Negating $\forall\,\exists$ gives $\exists\,\forall$: there is some x that is no cube. A negates only the inner clause; D over-negates. **C**
3. $-1-\sqrt{3}i$ lies in the third quadrant with reference angle $\frac{\pi}{3}$; the principal argument in $(-\pi,\pi]$ is $-\pi+\frac{\pi}{3}=-\frac{2\pi}{3}$ (B is the non-principal $\frac{4\pi}{3}$). **D**
4. Direction $\overrightarrow{AB}=(2,1,-2)$ through point A. A and B use a position vector as the direction; D uses $B-A$ as the *point*. **C**
5. $a=v\dfrac{dv}{dx}=(x^2+1)(2x)$; at $x=1$: $2\times 2=4$. (A is the omitted-v error $\frac{dv}{dx}=2$.) **B**
6. $v^2=16-(x-2)^2$: $n=1$, centre 2, amplitude 4. (B is the read-the-constant-term error $A^2=12$; D is the centre.) **A**
7. Conjugate $2-i$ is also a zero; sum of zeros $=7$, so the third is $7-4=3$ (or $15\div|2+i|^2=3$ from the product). **A**
8. Irreducible quadratic x^2+4 needs a linear numerator $Bx+C$; it does not factor over the reals (B), and $\frac{A}{x-1}$ needs only a constant (C). **D**
9. $1+i=\sqrt{2}\operatorname{cis}\frac{\pi}{4}$, so $(1+i)^{10}=2^5\operatorname{cis}\frac{5\pi}{2}=32\operatorname{cis}\frac{\pi}{2}=32i$. **B**
10. Equidistant from 1 and $-i$: the perpendicular bisector of the segment joining $(1,0)$ and $(0,-1)$, which passes through the origin with gradient -1: the line $y=-x$. **C**

Question 11

(a)(i) $\bar{z}w=(3+i)(1+2i)=3+6i+i-2=1+7i$.

(a)(ii) $\dfrac{3-i}{1+2i}\cdot\dfrac{1-2i}{1-2i}=\dfrac{3-6i-i-2}{5}=\dfrac{1-7i}{5}=\dfrac{1}{5}-\dfrac{7}{5}i$. (1 mark multiplying by the conjugate, 1 mark answer.)

(b) $|\sqrt{3}-i|=2$, $\arg=-\frac{\pi}{6}$, so $\sqrt{3}-i=2e^{-i\pi/6}$. Then $\left(\sqrt{3}-i\right)^7=2^7e^{-7i\pi/6}=128\operatorname{cis}\frac{5\pi}{6}=128\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=-64\sqrt{3}+64i$. (1 mark exponential form with principal argument, 1 mark De Moivre, 1 mark exact Cartesian.)

(c) $\underset{\sim}{a}\cdot\underset{\sim}{b}=2-3-4=-5$; $|\underset{\sim}{a}|=3$, $|\underset{\sim}{b}|=\sqrt{14}$. $\cos\theta=\dfrac{-5}{3\sqrt{14}}$, so $\theta\approx 116^\circ$. (Keep the negative cosine — the angle is obtuse.)

(d)(i) $\overrightarrow{AB} = (2,-1,2)$: $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$.

(d)(ii) Require $(7,-1,5) = (1+2\lambda,\ 2-\lambda,\ -1+2\lambda)$: the first gives $\lambda = 3$, the second $\lambda = 3$, the third $\lambda = 3$ — one **consistent** parameter, so C lies on ℓ. (The mark is for checking all three components against a single λ.)

(e) Suppose $\log_2 3 = \frac{p}{q}$ with p, q positive integers (positive since $\log_2 3 > 0$). Then $2^{p/q} = 3$, so $2^p = 3^q$. But 2^p is even and 3^q is odd — a contradiction. Hence $\log_2 3$ is irrational. (1 mark stating the assumption in lowest form, 1 mark reaching $2^p = 3^q$, 1 mark exhibiting the parity contradiction **and writing the conclusion**.)

(f) $z = \pm 3i$.

Question 12

(a)(i) $2x^2 + x + 2 = A(x^2+4) + (Bx+C)(x-1)$. At $x = 1$: $5 = 5A$, so $A = 1$. Comparing x^2: $B = 1$; comparing constants: $4A - C = 2$, so $C = 2$. Thus $A = 1$, $B = 1$, $C = 2$.

(a)(ii) $\displaystyle\int \left(\frac{1}{x-1} + \frac{x+2}{x^2+4}\right) dx = \ln|x-1| + \tfrac{1}{2}\ln\left(x^2+4\right) + \tan^{-1}\frac{x}{2} + C$. (Absolute value on the first logarithm required.)

(b) $dx = \dfrac{2\,dt}{1+t^2}$, $\sin x = \dfrac{2t}{1+t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$; limits $x = 0 \to t = 0$, $x = \frac{\pi}{2} \to t = 1$. The denominator becomes $\dfrac{2+2t}{1+t^2}$, so the integral is $\displaystyle\int_0^1 \frac{dt}{1+t} = \ln 2$. (1 mark substitution including **both limits**, 1 mark simplification, 1 mark $\ln 2$.)

(c)(i) $\ddot{x} = \frac{1}{2}\dfrac{d(v^2)}{dx} = \frac{1}{2}(18 - 18x) = -9(x-1)$, which is of the form $\ddot{x} = -n^2(x-c)$: SHM with centre $x = 1$ and $n = 3$, period $\dfrac{2\pi}{3}$.

(c)(ii) $v^2 = 36 - 9(x-1)^2$: amplitude $A = 2$; maximum speed $nA = 6$ m s^{-1} (at the centre).

(c)(iii) Maximum $|\ddot{x}| = n^2 A = 9 \times 2 = 18$ m s^{-2} (at the endpoints $x = -1, 3$).

(c)(iv) $4A = 8$ m.

(d) Base case $n = 5$: $2^5 = 32 > 25 = 5^2$. Assume $2^k > k^2$ for some integer $k \geq 5$. Then $2^{k+1} = 2 \cdot 2^k > 2k^2$. Now for $k \geq 5$, $k^2 - 2k - 1 = (k-1)^2 - 2 > 0$, so $2k^2 > k^2 + 2k + 1 = (k+1)^2$. Hence $2^{k+1} > (k+1)^2$, and the result follows by induction. (The middle mark is for the auxiliary inequality $2k^2 > (k+1)^2$ with justification valid for $k \geq 5$ — the step where the assumption is *used*, not restated.)

Question 13

(a)(i) With $u = x^n$, $dv = e^x dx$, so $du = nx^{n-1}dx$, $v = e^x$:

$$I_n = \left[x^n e^x\right]_0^1 - n\int_0^1 x^{n-1}e^x dx = e - nI_{n-1} \quad (n \geq 1).$$

(1 mark naming u, dv, v, du; 1 mark the parts step; 1 mark substituting the limits.)

(a)(ii) $I_0 = e - 1$; $I_1 = e - I_0 = 1$; $I_2 = e - 2I_1 = e - 2$; $I_3 = e - 3I_2 = 6 - 2e$.

(b) $|z-2| \leq 2$ is the closed disc centre $(2,0)$ radius 2 (solid boundary).
$|z| > |z - 2i| \iff x^2 + y^2 > x^2 + (y-2)^2 \iff y > 1$: the open half-plane above the **dashed** line $y = 1$. The region is the part of the disc strictly above $y = 1$; the circle meets $y = 1$ at $\left(2 \pm \sqrt{3},\ 1\right)$ (excluded endpoints, open dots). (1 mark disc with solid boundary, 1 mark dashed bisector $y = 1$ with correct side, 1 mark intersection points and correct shading.)

(c)(i) $(\sqrt{a} - \sqrt{b})^2 \geq 0 \Rightarrow a + b \geq 2\sqrt{ab} \Rightarrow \dfrac{a+b}{2} \geq \sqrt{ab}$.

(c)(ii) By (i), $a + b \geq 2\sqrt{ab}$, $b + c \geq 2\sqrt{bc}$, $c + a \geq 2\sqrt{ca}$. All six quantities are positive, so the inequalities may be multiplied:

$$(a+b)(b+c)(c+a) \geq 8\sqrt{a^2b^2c^2} = 8abc.$$

(1 mark applying (i) three times, 1 mark justifying the multiplication by positivity, 1 mark completing; equality iff $a = b = c$. Start from the known truth — do not work backwards from the target.)

(d) General point $Q(1 + 2\lambda,\ \lambda,\ -1 + 2\lambda)$; $\overrightarrow{PQ} = (2\lambda - 2,\ \lambda + 1,\ 2\lambda - 3)$. Perpendicularity: $\overrightarrow{PQ} \cdot (2,1,2) = 9\lambda - 9 = 0$, so $\lambda = 1$ and the closest point is $(3,1,1)$. Then $\overrightarrow{PQ} = (0,2,-1)$ and the shortest distance is $\sqrt{5}$. (1 mark general point, 1 mark perpendicularity condition, 1 mark foot and distance.)

Question 14

(a)(i) $\overrightarrow{OD} = \underset{\sim}{a} + \frac{2}{3}\left(\underset{\sim}{b} - \underset{\sim}{a}\right) = \frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$.

(a)(ii) X on OD: $\overrightarrow{OX} = t\left(\frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}\right)$. X on BC: $\overrightarrow{OX} = \underset{\sim}{b} + s\left(\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}\right) = \frac{s}{2}\underset{\sim}{a} + (1-s)\underset{\sim}{b}$. Since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are non-zero and **non-parallel**, coefficients may be equated: $\frac{t}{3} = \frac{s}{2}$ and $\frac{2t}{3} = 1 - s$. Solving: $t = \frac{3}{4}$, $s = \frac{1}{2}$. So $X = B + \frac{1}{2}(C - B)$ is the midpoint of BC, and $\overrightarrow{OX} = \frac{3}{4}\overrightarrow{OD}$ gives $OX : XD = 3 : 1$. (1 mark the two expressions, 1 mark equating coefficients **with the non-parallel justification**, 1 mark both conclusions.)

(b)(i) $\cos 5\theta + i \sin 5\theta = (\cos\theta + i\sin\theta)^5$. The imaginary part of the binomial expansion (with $c = \cos\theta$, $s = \sin\theta$): $\sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$. Substituting $c^2 = 1 - s^2$:

$$\sin 5\theta = 5s(1-s^2)^2 - 10s^3(1-s^2) + s^5 = 16s^5 - 20s^3 + 5s.$$

(b)(ii) For $\theta = \frac{\pi}{5}$ and $\theta = \frac{2\pi}{5}$, $\sin 5\theta = \sin\pi = \sin 2\pi = 0$, so $s\left(16s^4 - 20s^2 + 5\right) = 0$ with $s = \sin\theta \neq 0$; hence both values satisfy $16x^4 - 20x^2 + 5 = 0$.

(b)(iii) Putting $y = x^2$: $16y^2 - 20y + 5 = 0$ has roots $y_1 = \sin^2\frac{\pi}{5}$ and $y_2 = \sin^2\frac{2\pi}{5}$ (distinct, as $0 < \sin\frac{\pi}{5} < \sin\frac{2\pi}{5}$). By Vieta, $y_1 y_2 = \frac{5}{16}$, so $\left(\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\right)^2 = \frac{5}{16}$; both sines are positive, hence the product is $\frac{\sqrt{5}}{4}$. (The justification that the two roots of the quadratic are exactly these two values — and the positive-root choice — carries a mark.)

(c) Real coefficients $\Rightarrow 1-2i$ is also a zero. Sum of zeros $=5$: third zero $=5-(1+2i)-(1-2i)=3$. (Check: $(1+4)\times 3=15$.) Other zeros: $1-2i$ and 3.

(d) Discriminant: $(3+i)^2-4(2+2i)=8+6i-8-8i=-2i$. Seek $(x+iy)^2=-2i$: $x^2-y^2=0$, $2xy=-2$, giving $x=1, y=-1$ (or $x=-1, y=1$), so $\sqrt{-2i}=\pm(1-i)$. Then

$$z=\frac{(3+i)\pm(1-i)}{2}=2 \quad \text{or} \quad 1+i.$$

(1 mark discriminant, 1 mark Cartesian square root, 1 mark both roots. Check: sum $=3+i$, product $=2+2i$.)

Question 15

(a)(i) $v\dfrac{dv}{dx}=-(g+kv^2)$, so $x=-\displaystyle\int\frac{v\,dv}{g+kv^2}=-\frac{1}{2k}\ln\left(g+kv^2\right)+C$. At $x=0$, $v=u$; at the top $v=0$:

$$H=\frac{1}{2k}\ln\frac{g+ku^2}{g}=\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right).$$

(1 mark separating with the correct acceleration form, 1 mark the log integral with its constant, 1 mark applying both conditions to reach the given form — show every step in a "show that".)

(a)(ii) $\dfrac{dv}{dt}=-(g+kv^2)$, so

$$T=\int_0^u\frac{dv}{g+kv^2}=\frac{1}{k}\cdot\frac{1}{\sqrt{g/k}}\left[\tan^{-1}\frac{v}{\sqrt{g/k}}\right]_0^u=\frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right).$$

(The question is sequenced to force **both** acceleration forms — the marking-centre theme of 2020, 2021, 2023, 2024, 2025.)

(a)(iii) Falling: $\ddot{x}=g-kv^2$ (resistance now opposes the downward motion), so the terminal velocity is $v_T=\sqrt{g/k}$, where the acceleration vanishes. While falling, $v<v_T$ always: the speed increases towards v_T but the acceleration $g-kv^2\to 0$ as $v\to v_T$, so v_T is approached asymptotically and never attained in the finite fall from height H. Hence the landing speed is less than $\sqrt{g/k}$.

(b)(i) $|-8+8\sqrt{3}i|=16$, $\arg=\frac{2\pi}{3}$. The fourth roots have modulus $16^{1/4}=2$ and arguments $\frac{2\pi/3+2k\pi}{4}=\frac{\pi}{6}+\frac{k\pi}{2}$:

$$z=2e^{i\pi/6},\quad 2e^{2i\pi/3},\quad 2e^{-5i\pi/6},\quad 2e^{-i\pi/3}.$$

(b)(ii) Four points on the circle $|z|=2$, equally spaced at right angles — the vertices of a square centred at O.

(b)(iii) $2e^{i\pi/6}=2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\sqrt{3}+i$.

(c) (i) $\bar{z}$: reflect z in the real axis — fourth quadrant, same modulus (outside the unit circle). (ii) $i\bar{z}$: rotate $\bar{z}$ anticlockwise by $\frac{\pi}{2}$ — argument $\frac{\pi}{2}-\arg z\approx 50^\circ$, same modulus. (iii) $\dfrac{z^2}{|z|}$: argument doubles to

$\approx 80^\circ$, modulus $\dfrac{|z|^2}{|z|} = |z|$ — the same distance from O as z, rotated to twice the argument. (1 mark each; the marks are for correct modulus **and** argument reasoning, not computation.)

Question 16

(a)(i) $|z+w|^2 + |z-w|^2 = (z+w)(\bar{z}+\bar{w}) + (z-w)(\bar{z}-\bar{w}) = 2z\bar{z} + 2w\bar{w} = 2|z|^2 + 2|w|^2$ (the cross terms $z\bar{w} + w\bar{z}$ cancel).

(a)(ii) With $|z| = |w| = 1$, part (i) gives $|z+w|^2 + |z-w|^2 = 4$. Suppose, for contradiction, that $|z+w| < \sqrt{2}$ **and** $|z-w| < \sqrt{2}$. Then $|z+w|^2 + |z-w|^2 < 2 + 2 = 4$ — contradicting the identity. Hence at least one of them is $\geq \sqrt{2}$. (1 mark the assumption stated, 1 mark the contradiction exhibited and the conclusion written.)

(a)(iii) $z = 1$, $w = i$: $|1+i| = |1-i| = \sqrt{2}$, and $|z| = |w| = 1$. (Any perpendicular pair on the unit circle works.)

(b)(i) $v\dfrac{dv}{dx} = -(c+kv)$, so

$$D = \int_0^u \frac{v\,dv}{c+kv} = \int_0^u \left(\frac{1}{k} - \frac{c/k}{c+kv}\right)dv = \frac{u}{k} - \frac{c}{k^2}\ln\frac{c+ku}{c} = \frac{u}{k} - \frac{c}{k^2}\ln\left(1 + \frac{ku}{c}\right).$$

(1 mark the correct acceleration form and separation, 1 mark the division/decomposition of $\frac{v}{c+kv}$, 1 mark limits to the given form.)

(b)(ii) $\dfrac{dv}{dt} = -(c+kv)$: $T = \displaystyle\int_0^u \frac{dv}{c+kv} = \frac{1}{k}\ln\left(1 + \frac{ku}{c}\right)$.

(c)(i) With $u = \pi - x$ (so $\sin x = \sin u$, $\cos x = -\cos u$, $dx = -du$, limits swap):

$$I = \int_0^\pi \frac{(\pi - u)\sin u}{1+\cos^2 u}\,du = \pi\int_0^\pi \frac{\sin u}{1+\cos^2 u}\,du - I,$$

so $2I = \pi\displaystyle\int_0^\pi \frac{\sin x}{1+\cos^2 x}\,dx$. (1 mark the substitution with both limits, 1 mark collecting I.)

(c)(ii) Let $c = \cos x$, $dc = -\sin x\,dx$: $\displaystyle\int_{-1}^1 \frac{dc}{1+c^2} = \left[\tan^{-1} c\right]_{-1}^1 = \frac{\pi}{2}$.

(c)(iii) $I = \dfrac{\pi}{2}\cdot\dfrac{\pi}{2} = \dfrac{\pi^2}{4}$.

Prediction provenance

Which consensus prediction each part of this paper realises. Full evidence panels: intuitioneducation.com.au/hsc-2026

Paper item	Prediction	Agreement
MC Q1, Q2 contrapositive + quantifier negation	`ext2-q1-logic-mc`	0.82 · 6 (all)
Q11(a), (b), (f) conjugate arithmetic, realised quotient, exponential-form 7th power	`ext2-q2-q11-complex-opener`	0.81 · 5 (fable, opus, grok, deepseek, gpt-5.6-sol)
Q12(a) + MC Q8 partial fractions, $(x-p)(x^2+q)$ → ln + arctan	`ext2-q3-partial-fractions`	0.78 · 6 (all)
Q12(c) + MC Q6 SHM from v^2 quadratic, shifted centre; amplitude-off-constant-term distractor	`ext2-q4-shm`	0.76 · 6 (all)
Q13(a) reduction formula by parts with "hence" evaluation	`ext2-q5-reduction-formula`	0.71 · 6 (all) — realised as the predicted reversion to integration by parts
Q11(c), (d) + MC Q4 angle between vectors, line through two points, point-on-line test	`ext2-q6-vector-line-toolkit`	0.71 · 5 (fable, opus, grok, gpt-5.6-sol, deepseek)
Q13(c) AM–GM prove-then-apply, chained 3-variable inequality	`ext2-q7-am-gm`	0.66 · 5 (opus, fable, grok, gpt-5.6-sol, deepseek)
Q13(b) Argand region: bisector inequality ∩ disc, dashed/solid conventions	`ext2-q8-argand-region`	0.64 · 6 (all)
Q15(a) vertical resisted motion kv^2, sequenced to force both acceleration forms	`ext2-q9-resisted-vertical`	0.64 · 5 (deepseek, gpt-5.6-sol, grok, fable + opus variant)
Q11(e) contradiction: irrationality of $\log_2 3$ (logical frame marked)	`ext2-q10-contradiction-irrationality`	0.63 · 6 (all) — type consensus; fresh surface constant
Q14(a) vector ratio proof by equating coefficients of non-parallel vectors	`ext2-q11-vector-ratio-proof`	0.62 · 5 (fable, opus, grok, gpt-5.6-sol, deepseek)
Q12(b) $t=\tan\frac{x}{2}$ supplied-substitution integral; Q16(c) the $u=a-x$ symmetry variant	`ext2-q12-substitution-integral`	0.60 · 5 — both panel sub-variants realised
Q14(b) De Moivre → $\sin 5\theta$ polynomial → exact surd product	`ext2-q13-de-moivre-exact-value`	0.58 · 5 — fresh surface (sine identity) per the corpus-echo note on cos 5θ
Q12(d) inequality induction with base case $n=5$, auxiliary inequality cited	`ext2-q14-induction-slot`	0.52 · 6 (all, type-level) — inequality form favoured over the divisibility outsider; the DeepSeek/Grok shared surface (7^n-2^n) was a training-data echo and was **not** used
Q13(d) shortest distance from point to line via foot of perpendicular	`watch list`	~0.63 · 3 (deepseek, opus, gpt-5.6-sol) — the panel's sharpest split: fable and grok rest it
Q15(b) $z^4=c$ for non-real c, equally spaced roots, Argand plot	`watch list`	~0.60 · 3 — fresh surface (2025 used $z^5+1=0$)

Paper item	Prediction	Agreement
Q16(b) horizontal resisted motion, constant-plus-v, distance-to-rest log form	`watch list`	~0.63 · 2 (opus, fable)
MC Q5 $a = v\,dv/dx$ with the omitted-v distractor	`watch list`	~0.60 · 2 (grok, gpt-5.6-sol) + deepseek trend
Q14(c) + MC Q7 conjugate-root polynomial, remaining zeros by sum/product	`watch list`	~0.54 · 3 (fable, gpt-5.6-sol, grok)
Q14(d) complex-coefficient quadratic via Cartesian $\sqrt{\Delta}$	`watch list`	~0.48 · 3 (deepseek, fable, opus) — fable's bold call: five-year Section II coverage gap in the syllabus's final year
Q16(a) abstract bound proof: parallelogram identity + contradiction on moduli	`watch list`	0.65 · fable (specific + structural; 2021 16(a), 2022 15(d), 2025 16(a) lineage)
Q15(c) Argand transformation plot: $\bar{z}$, $i\bar{z}$, $z^2/\lvert z\rvert$ without computation	`watch list`	~0.55 · 2 (fable, gpt-5.6-sol)
MC Q3 principal argument; MC Q9 De Moivre power; MC Q10 bisector locus	`topic-level consensus`	— · —
Not realised: resisted projectile (rested by fable and grok after 2025 16(b)); volumes-of-revolution revival (grok alone, 0.48)	`topic-level consensus`	— · —

Built by an AI panel and backtested against the hidden 2025 papers before publication — method at intuitioneducation.com.au/hsc-2026/how-we-did-it. Scored publicly in November 2026. Independent Intuition Education material from publicly available NESA documents; not affiliated with or endorsed by NESA.