INTUITION EDUCATION

HSC 2026 · PREDICTED PRACTICE PAPER

2026 HSC Mathematics Extension 1

Answers & marking notes

Paper total: 70 marks

Sit the paper first. These are worked answers with the marking notes behind them — what earns the mark, and the traps the marking centre has flagged in past years. Where a note names a band, it describes what a top-band response does, not a guaranteed mark.

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Answers & marking notes

Section I — correct responses

Q	1	2	3	4	5	6	7	8	9	10
Answer	C	C	B	A	C	A	D	B	D	B

Brief reasons:

1. $\dfrac{8!}{3!\,2!} = \dfrac{40320}{12} = 3360$ (three L's, two A's). **C**
2. $\underset{\sim}{u} \cdot \underset{\sim}{v} = 3 - 2 = 1$, $|\underset{\sim}{v}|^2 = 5$, so $\frac{1}{5}(\underset{\sim}{i} + 2\underset{\sim}{j})$. Distractors: A divides by $|\underset{\sim}{u}|^2$ *and* attaches the scalar to $\underset{\sim}{u}$; B attaches to the wrong vector; D divides by $|\underset{\sim}{u}|^2 = 10$. **C**
3. $\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$; the principal value in $[0, \pi]$ is $\frac{3\pi}{4}$. **B**
4. $\dfrac{1 - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}} = \dfrac{2t^2}{2t} = t$. **A**
5. $\sin x\,(2\cos x - 1) = 0$: $x = 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ — five solutions. Dividing by $\sin x$ would incorrectly discard $x = 0, \pi, 2\pi$. **C**
6. $\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$, so $\dfrac{dr}{dt} = \dfrac{100}{4\pi(25)} = \dfrac{1}{\pi}$. **A**
7. $\sigma_{\hat{p}} = \sqrt{\dfrac{0.2 \times 0.8}{100}} = \sqrt{0.0016} = 0.04$. **D**
8. $\binom{7}{4} \cdot 2^3 = 35 \times 8 = 280$. **B**
9. $f(1) = 5$, $f'(1) = 6$, so $(f^{-1})'(5) = \frac{1}{f'(1)} = \frac{1}{6}$. **D**
10. Seven days of the week (holes); $7 \times 3 = 21$ people can avoid a group of 4, so 22 guarantees it. **B**

Question 11

(a)(i) $2\underset{\sim}{a} - \underset{\sim}{b} = (4+1)\underset{\sim}{i} + (6-4)\underset{\sim}{j} = 5\underset{\sim}{i} + 2\underset{\sim}{j}$.

(a)(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b} = -2 + 12 = 10$.

(b) $P(1) = 0$: $2 + a + b - 3 = 0 \Rightarrow a + b = 1$. $P(-2) = -3$: $-16 + 4a - 2b - 3 = -3 \Rightarrow 2a - b = 8$. Adding: $3a = 9$, so $a = 3$, $b = -2$. (1 mark each equation, 1 mark solving.)

(c) Multiply both sides by $x^2 > 0$ ($x \neq 0$): $3x \leq x^3 + 2x^2$, i.e. $x(x+3)(x-1) \geq 0$. Sign analysis of the cubic gives $-3 \leq x < 0$ or $x \geq 1$ ($x = 0$ excluded as it zeroes the denominator). (1 mark valid method, 1 mark three critical values $-3, 0, 1$, 1 mark correct union with $x = 0$ excluded.)

(d)(i) $R = \sqrt{3+1} = 2$, $\tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = \frac{\pi}{6}$; so $2\sin\left(x - \frac{\pi}{6}\right)$.

(d)(ii) $\sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$ with $x - \frac{\pi}{6} \in \left[-\frac{\pi}{6}, \frac{11\pi}{6}\right]$: $x - \frac{\pi}{6} = \frac{\pi}{4}, \frac{3\pi}{4}$, so $x = \frac{5\pi}{12}, \frac{11\pi}{12}$.

(e) $du = 2x\,dx$; limits $x = 0 \to u = 9$, $x = 4 \to u = 25$.

$\frac{1}{2}\int_9^{25} u^{1/2} du = \frac{1}{3}\left[u^{3/2}\right]_9^{25} = \frac{125-27}{3} = \dfrac{98}{3}$.

Question 12

(a) Base case $n = 1$: $5 + 22 = 27 = 3 \times 9$. Assume $5^k + 2 \times 11^k = 3M$, M an integer. Then $5^{k+1} + 2 \times 11^{k+1} = 5\left(5^k + 2 \times 11^k\right) + 12 \times 11^k = 3\left(5M + 4 \times 11^k\right)$, divisible by 3. Conclusion by induction. (The middle mark requires the assumption to be *substituted* — e.g. via the regrouping above or by writing $5^k = 3M - 2 \times 11^k$ — not merely restated.)

(b) $y\,dy = x\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$. At $(3, -4)$: $16 = 9 + C' \Rightarrow y^2 = x^2 + 7$. Since $y(3) = -4 < 0$ and $y \neq 0$, the solution stays on the negative branch: $y = -\sqrt{x^2 + 7}$. (1 mark separation/integration, 1 mark constant, 1 mark negative branch with justification.)

(c)(i) An increasing S-shaped (logistic-type) curve through $(0, 0.5)$, following the slope segments, concave up then concave down, approaching the asymptote $y = 2$ from below as x increases (never crossing it, never crossing $y = 0$).

(c)(ii) $y = 2$ is itself a (constant) solution along which $\dfrac{dy}{dx} = 0$; solution curves cannot cross it, so a curve starting below $y = 2$ remains below it and can never reach $B(3, 2.5)$.

(d) $x^2 = \dfrac{4}{y} - 1$; curve meets y-axis at $y = 4$ and the line at $y = 1$.

$$V = \pi \int_1^4 \left(\frac{4}{y} - 1\right) dy = \pi[4\ln y - y]_1^4 = \pi\left(4\ln 4 - 3\right) = (8\ln 2 - 3)\pi \text{ cubic units.}$$

(e) $\cos 2x = 2\cos^2 x - 1$: equation becomes $\cos x\left(2\cos x + 1\right) = 0$. $\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$; $\cos x = -\frac{1}{2}$: $x = \frac{2\pi}{3}, \frac{4\pi}{3}$. (Factorise — do not divide by $\cos x$.)

Question 13

(a) Decreasing curve from endpoint $(-2, 3\pi)$ to endpoint $(2, 0)$, passing through $\left(0, \frac{3\pi}{2}\right)$; domain $[-2, 2]$, range $[0, 3\pi]$.

(b) $\overrightarrow{PA} = \underset{\sim}{a} - \underset{\sim}{p}$ and $\overrightarrow{PB} = -\underset{\sim}{a} - \underset{\sim}{p}$. Then

$\overrightarrow{PA} \cdot \overrightarrow{PB} = (\underset{\sim}{a} - \underset{\sim}{p}) \cdot (-(\underset{\sim}{a} + \underset{\sim}{p})) = -\left(|\underset{\sim}{a}|^2 - |\underset{\sim}{p}|^2\right) = -(r^2 - r^2) = 0$ since $|\underset{\sim}{a}| = |\underset{\sim}{p}| = r$ (radii). Hence $\overrightarrow{PA} \perp \overrightarrow{PB}$, i.e. $\angle APB = 90^\circ$.

(c)(i) $\mu = np = 360$; $\sigma = \sqrt{np(1-p)} = \sqrt{36} = 6$.

(c)(ii) $P(X < k) = 0.0668 \Rightarrow P\left(Z < \dfrac{k - 360}{6}\right) = 1 - 0.9332$, so $\dfrac{k - 360}{6} = -1.5$ and $k = 360 - 9 = 351$. (Reverse use of the table: 1 mark for $z = -1.5$ via symmetry, 1 mark standardising, 1 mark $k = 351$.)

(d) $\sin^2 2x = \frac{1}{2}(1-\cos 4x)$ (reference sheet):

$$\left[\frac{x}{2} - \frac{\sin 4x}{8}\right]_0^{\pi/8} = \frac{\pi}{16} - \frac{1}{8}.$$

(e) $\alpha+\beta+\gamma = -2$, $\alpha\beta+\beta\gamma+\gamma\alpha = -3$. So $\alpha^2+\beta^2+\gamma^2 = (-2)^2 - 2(-3) = 10$.

Question 14

(a) $x^2+2x+5 = (x+1)^2+4$, so the integral is $\left[\frac{1}{2}\tan^{-1}\frac{x+1}{2}\right]_{-1}^{1} = \frac{1}{2}\left(\tan^{-1}1 - \tan^{-1}0\right) = \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$.

(b) Join the midpoints of the sides: this divides the triangle into four equilateral triangles of side 1 cm (construct the holes). By the pigeonhole principle, two of the five points lie in the same small triangle; any two points in an equilateral triangle of side 1 are at most 1 cm apart (its diameter is its side). Hence two points are no more than 1 cm apart.

(c)(i) If $\tan 3\theta = 1$ with $t = \tan\theta$: $3t - t^3 = 1 - 3t^2$, i.e. $t^3 - 3t^2 - 3t + 1 = 0$. $\tan 3\theta = 1$ for $3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$, i.e. $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$, giving three distinct tan values — the three roots of the cubic.

(c)(ii) Sum of roots $= 3$: $\tan\frac{\pi}{12} + \tan\frac{5\pi}{12} + \tan\frac{3\pi}{4} = 3$, and $\tan\frac{3\pi}{4} = -1$, so $\tan\frac{\pi}{12} + \tan\frac{5\pi}{12} = 4$.

(d)(i) A lands when $\frac{\sqrt{3}Vt}{2} - 5t^2 = 0$, $t > 0$: $t = \frac{\sqrt{3}V}{10}$. For B, with flight time $s = t - T$: $\frac{Vs}{2} - 5s^2 = 0$, $s > 0$: $s = \frac{V}{10}$. Ranges: $x_A = \frac{V}{2}\cdot\frac{\sqrt{3}V}{10} = \frac{\sqrt{3}V^2}{20}$ and $x_B = \frac{\sqrt{3}V}{2}\cdot\frac{V}{10} = \frac{\sqrt{3}V^2}{20}$ — equal, so same landing point. (Key trap: the two balls have *different* flight-time variables; each vertical displacement is set to zero separately.)

(d)(ii) Same instant: $T + \frac{V}{10} = \frac{\sqrt{3}V}{10}$, so $T = \frac{(\sqrt{3}-1)V}{10}$. With $T = 2$: $V = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3}+1)\ \mathrm{m\,s^{-1}}$.

(d)(iii) $\dot{\underset{\sim}{r}}_A$ at landing: $\left(\frac{V}{2}, \frac{\sqrt{3}V}{2} - 10\cdot\frac{\sqrt{3}V}{10}\right) = \left(\frac{V}{2}, -\frac{\sqrt{3}V}{2}\right)$ — direction 60° below horizontal. $\dot{\underset{\sim}{r}}_B$ at landing: $\left(\frac{\sqrt{3}V}{2}, -\frac{V}{2}\right)$ — direction 30° below horizontal. The acute angle between them is $60^\circ - 30^\circ = 30^\circ = \frac{\pi}{6}$. (Equivalently via $\cos\phi = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{|\underset{\sim}{u}||\underset{\sim}{v}|} = \frac{\sqrt{3}}{2}$.)

Prediction provenance

Which consensus prediction each part of this paper realises. Full evidence panels: intuitioneducation.com.au/hsc-2026

Paper item	Prediction	Agreement
Q11(e) given-substitution integral (and Q14(a) arctan variant)	`ext1-q1-substitution-integral`	0.76 · 5 (grok, fable, opus, gpt-5.6-sol, deepseek)
Q14(d) two-ball staggered projectile, $r(t)$ supplied	`ext1-q2-projectile-twist`	0.64 · 6 (all)
Q11(d) auxiliary-angle express-and-solve	`ext1-q3-auxiliary-angle`	0.67 · 6 (all)
Q12(a) 3-mark divisibility induction (rearranged assumption)	`ext1-q4-divisibility-induction`	0.61 · 6 (all)
Q12(b) separable DE with branch choice	`ext1-q5-separable-de`	0.69 · 6 (all)
Q12(c) direction field: sketch vs asymptote + justification	`ext1-q6-direction-field`	0.61 · 6 (all)
Q12(d) volume of revolution about the y-axis (π + log)	`ext1-q7-volume-y-axis`	0.62 · 4 (deepseek, gpt-5.6-sol, fable, opus)
Q13(c) reverse normal approximation with z-table	`ext1-q8-normal-approximation`	0.64 · 5 (grok, deepseek, opus, gpt-5.6-sol, fable)
Q11(c) rational inequality, critical values + exclusion	`ext1-q9-rational-inequality`	0.58 · 6 (all)
MC Q2 projection with the three classic distractors	`ext1-q10-projection`	0.66 · 5 (gpt-5.6-sol, gemini-3.1-pro, deepseek, fable, opus)
Q11(b) factor + remainder theorem, unknown coefficients	`ext1-q11-factor-remainder`	0.57 · 6 (all) — type only; fresh constants per contamination_note (the DeepSeek/Grok surface constants were a shared-training-data echo and were **not** used)
MC Q9 derivative of an inverse function at a point	`ext1-q12-inverse-derivative`	0.52 · 4 (gpt-5.6-sol, deepseek, fable, opus)
Q13(d) $\sin^2(kx)$ definite integral via double-angle	`watch list`	~0.56 · 5
Q13(b) vector dot-product perpendicularity proof (circle)	`watch list`	~0.52 · 4
Q12(e) + MC Q5 double-angle equation; factorise, don't divide	`watch list`	~0.55 · 4
Q13(e) sum/product of roots symmetric expression	`watch list`	~0.55 · 4
Q14(b) pigeonhole, construct-the-holes; MC Q10 guarantee	`watch list`	0.55 / 0.58 · opus / grok variants

Paper item	Prediction	Agreement
form		
MC Q1 word arrangements with repeated letters	`watch list`	~0.54 · 4
Q13(a) inverse-trig sketch $y = a\cos^{-1}(bx)$ with endpoints	`watch list`	~0.54 · 4
MC Q8 binomial-expansion coefficient (coverage gap)	`watch list`	0.50 · fable
MC Q4 t-formula simplification	`topic-level consensus`	0.75 (single model) · gemini-3.1-pro
MC Q3, MC Q6, MC Q7, Q14(c)	`topic-level consensus`	— · —

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